Quantum Signal Processing via QSVT

A spectral view of quantum algorithms

Low-degree QSP/QSVT study with four Hermitian examples, a gap sweep, and a finite-shot comparison

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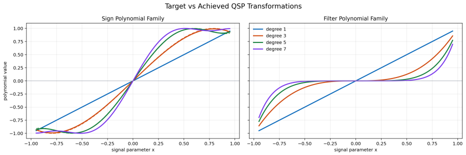
Thesis

QSP and QSVT provide one common language for several quantum algorithm ideas: they implement polynomial transformations on spectra.



Why QSP/QSVT matters

- Many quantum algorithms can be reframed as spectral transformations rather than unrelated circuit tricks.
- QSP gives scalar polynomial control through a short sequence of phase rotations.
- QSVT extends that same idea to block-encoded matrices.

Guiding question

What function do we want the circuit to apply to the spectrum?

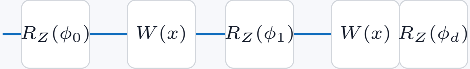
Roadmap: scalar QSP → block encoding → experimental results

QSP mechanics

$$W(x) = \begin{bmatrix} x & i\sqrt{1-x^2} \\ i\sqrt{1-x^2} & x \end{bmatrix}$$

This single-qubit signal operator encodes the scalar parameter x in its eigenphase structure.

Alternating $W(x)$ with Z-axis phases produces a bounded low-degree response in x.



Interpretation

The phases $\phi_0, \ldots, \phi_d$ are the tunable program. The resulting $(1,1)$ entry acts like the polynomial response.

From QSP to QSVT and study design

$$U_H = \begin{bmatrix} H & i\sqrt{I - H^2} \\ i\sqrt{I - H^2} & H \end{bmatrix}$$

$$H \in \{H_1, \ldots, H_4\}$$

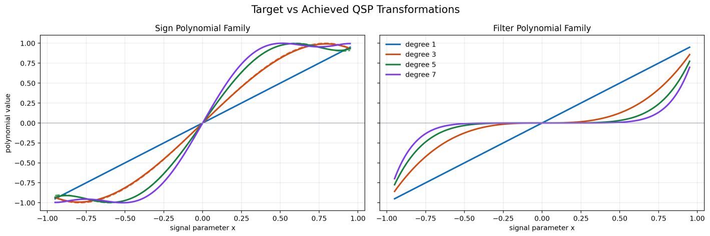
What block encoding does

It embeds a small Hermitian matrix into a larger unitary so the QSP phase-sequence logic can act on its eigenvalues.

- two polynomial families: sign-style and filter
- four 2×2 Hermitian examples: diagonal, rotated, small-gap, balanced-mix
- degrees $1, 3, 5, 7$
- sign-gap sweep: $\delta \in \{0.20, 0.35, 0.50\}$
- one finite-shot comparison for the degree-7 sign warm-up

So the larger experiment still keeps one coherent idea: the same phase-sequence logic is being reused across spectra, degrees, and approximation difficulty.

Scalar experiment results

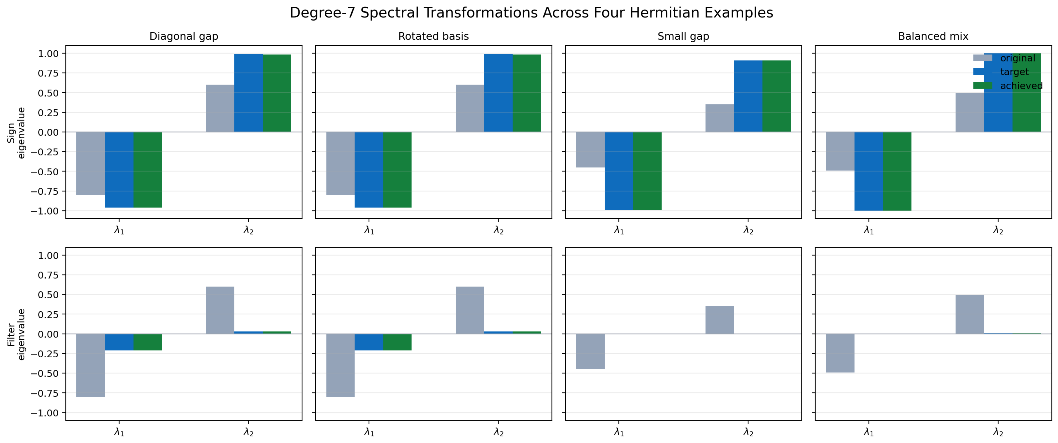


Two families

- **Filter:** x, x^3, x^5, x^7
- **Sign-style:** bounded odd approximation to $\mathrm{sign}(x)$ outside a gap

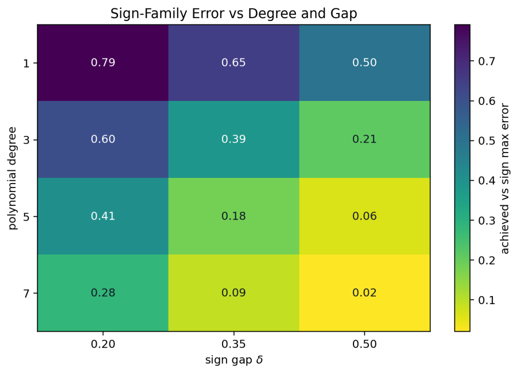
Degree 7 gives the sharpest sign-like transition, while the filter family remains essentially exact across the low-degree study.

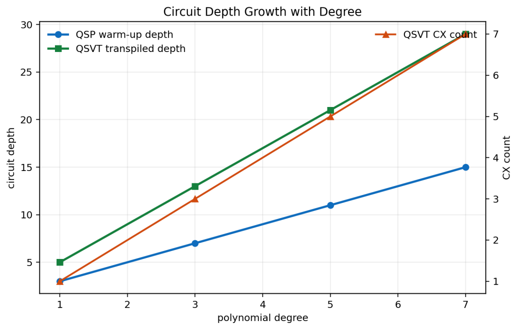
Spectral transformation results across four matrices



At degree 7, the sign family pushes spectra toward ± 1, while the filter family suppresses magnitude without changing the underlying spectral ordering.

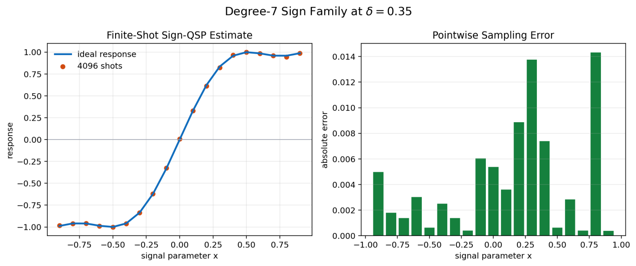
Tradeoffs: degree, gap, and circuit cost





- at $\delta = 0.35$, sign error drops $0.65 \to 0.39 \to 0.18 \to 0.09$
- QSVT depth grows $5 \to 13 \to 21 \to 29$
- QSVT CX count grows $1 \to 3 \to 5 \to 7$
- larger gap and higher degree both help, but sharper shaping costs circuit resources

Finite-shot check and conclusions



Takeaways

- QSP/QSVT is best understood as a spectral framework.
- The scalar and matrix cases share the same phase-sequence logic.
- The expanded study still stays interpretable: four matrices, a gap sweep, and one finite-shot check.

Limitations: not a general phase compiler, only a finite-shot check, and still restricted to tiny examples.

Grover as an intuition bridge

- Grover evolves inside a two-dimensional subspace of "good" and "bad" amplitudes.
- Repeating the iterate amplifies the desired component through a structured rotation.
- QSP broadens that idea: phase choices shape an odd polynomial response.

Amplitude amplification is the simplest example of the broader phase-programming viewpoint.

Key analogy

$$\text{Grover iterate} \implies \text{odd polynomial action}$$

Takeaway
The same rotation intuition scales from search-style amplification to more general spectral transformations.